Analytical Calculation of Magnetic Field Distribution in Vernier Machines with Doubly Salient Structure

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This paper presents an analytical method for the armature magnetic field distribution of vernier machines with a doubly salient structure. The approach to calculate the armature magnetic field is to determine the air-gap permeance and winding magneto-motive (MMF) distributions. The air-gap permeance function which includes the stator and rotor slotting effects is proposed. Then, the winding MMF distribution is predicted by using the permeance function and the simple equivalent magnetic circuit. For the vernier machine with the consequent pole rotor, the validity of the proposed method is verified by the finite element analysis method. In the analytical method, the harmonic components of the magnetic field are given as the function including the dimensional variable for the doubly salient structure. Thus, it is useful to determine the dimensions for the doubly salient structure based on the harmonic characteristics of the armature magnetic field.

***Index Terms*— Armature, electromagnetic devices, magnetic circuits, magnetic flux density,**

I. Introduction

RECENTLY, industrial demands are increasing for electric machines with high torque density and high efficiency. Hence, vernier machines that exploit the magnetic gearing effects are getting attention due to their high torque capability. In the vernier machine, the armature magnetic field is modulated by the geometries of the stator and rotor cores. Then, the modulated magnetic field generates the output torque on the rotor. Thus, in the vernier machine, it is more important to analyze and design the armature magnetic field with considering the configurations of the armature windings and the structures of the stator and rotor core.

The vernier machine was originally proposed as a kind of the reluctance machines about 50 years ago [1]. Then, with employing the PMs, the many topologies of the permanent magnet vernier (PMV) machines have been proposed, such as the surface-mounted PM (SPM) and consequent pole, etc [2]. Although the many topologies are proposed, the vernier machines can be classified into two categories according to the rotor structure with or without salient poles. The SPMV machine has the stator salient poles and the rotor core without the salient poles. In contrast, the other topologies of the vernier machines have the doubly salient poles on both sides of the stator and rotor. For the SPMV machine, the analysis and design method for the armature magnetic field is developed by using the air-gap permeance method [3]. However, in the vernier machine with the doubly salient structure, it is difficult to analyze the armature magnetic field in the same manner with the SPMV machine because the winding MMF and air-gap permeance distributions are varied according to the positions of the rotor. This paper presents the analytical method to predict the armature magnetic field in the vernier machine with the doubly salient structure. The analytical expressions for the permeance, winding MMF, and flux density in the air-gap are derived by using the Fourier analysis technique. The validity of the analytical method is verified by the finite element analysis (FEA) method. The harmonic components of the armature magnetic field are given as the functions of the design variables for the doubly salient poles. Thus, the analytical method can be used to determine the dimensions of the doubly salient structure for improving the working harmonic of the armature magnetic field, which generates output torque on the rotor of the vernier machine.

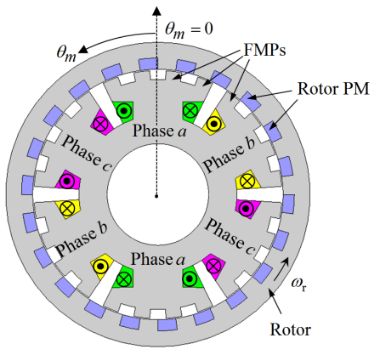

Fig. 1. A structure of the vernier machine with the consequent pole rotor.

II. Armature Magnetic field

Fig. 1 shows the structure of the vernier machine with the consequent pole rotor [2]. The armature windings of the three phases produce the magnetic field of 4 poles. The number of the salient poles on the stator and rotor are 24 and 22, respectively. In addition, The PMs are embedded on the side of the rotor, and all of the PMs are magnetized in the inward radial direction.

A. Air-gap permeance function

When assuming that that stator and rotor cores have infinite permeability, the permeance per unit area in the air-gap can be determined by the length of the magnetic flux path [3]. Fig. 2 shows the air-gap permeance distribution by the stator and rotor slots. In addition, the air-gap permeance function can be given as

$$P(\theta_m, \omega_r t) = P_s(\theta_m) \times P_r(\theta_m, \omega_r t) \tag{1}$$

where θ_m and ω_r are the mechanical angles and the angular speed of the rotor, respectively. $P_s(\theta_m)$ is the permeance function when there are no rotor slots [3], and $P_r(\theta_m, \omega_r t)$ is the relative permenance function which means the variation ratio of the length of the magnetic flux path due to the rotor slotting effects. P, P_s, and P_r can be expressed as the Fourier series, and their Fourier coefficients are given as the function of the dimensional parameters for the doubly salient structure.

B. *Winding MMF distribution*

Using the Ampere's law in combination with Gauss's law, the equivalent magnetic circuit and MMF distribution by the phase a winding is obtained as shown in Fig. 3. The maximum and minimum values of the MMF distribution depend on the values of the magnetic reluctance per slot pitch, R_1, R_2, R_3 as shown in Fig. 3. Using (1), the magnetic reluctance per slot pitch can be calculated as following.

$$R = l / \int_{\tau_p} P(\theta_m, \omega_r t) \cdot r \cdot d\theta_m \tag{2}$$

where r, l, and τp are the radius of the effective air-gap, the stacking length, and the slot pitch, respectively. Based on the (2), R_1, R_2, and R_3 are calculated according to the rotor positions. Then, the max and min values of the MMF are given as $q_{a1}{\cdot}N_c{\cdot}i_a$ and $q_{a2}{\cdot}N_c{\cdot}i_a$, respectively. Where, N_c is the number of turns per the tooth, and i_a is the instantaneous value of the phase a current. In addition, q_{a1} and q_{a2} are defined as $R_1/(R_1+R_2//R_3)$ and $(R_2//R_3)/(R_1+R_2//R_3)$, respectively.

The winding MMF distribution by the phase a winding shown in Fig. 3 (c) can be expressed as

$$f_a(\theta_m) = F_0 + \sum_{m=1,2,3,\cdots}^{\infty} F_m^a \cos(mp_s\theta_m) \tag{3}$$

$$\begin{cases} F_0 = \dfrac{N_c i_a}{3}(q_{a1} - 2q_{a2}) \\ F_m^a = \dfrac{4N_c i_a}{m^2 p_s \theta_s \pi} \sin\left(\dfrac{m\pi}{3}\right) \sin\left(\dfrac{mp_s\theta_s}{2}\right) \end{cases} \tag{4}$$

where p_s is the numbers of the stator pole pairs, θ_s is the mechanical angle of the stator slot opening. F_0 is the DC offset value and is changed according to the rotor positions. The MMF distributions by the phase b and c can be found in the same manner. Hence, the total MMF distribution is calculated by the sum of the produced MMF distribution by each phase.

C. *Radial flux density distribution by the stator windings*

The radial flux density distribution by the armature winding can be calculated by multiplying the air-gap permeance and winding MMF distributions. Fig. 4 shows the radial flux density distribution and their harmonic components that obtained by the analytical and FEA methods. In the both method, the difference of the flux density distribution result from the modeling method of the relative permeance function, P_r in the regions between the two rotor salient poles. However, it can be seen that the harmonic components have the similar trend in the analytical and FEA results. Moreover, in the analytical method, the harmonic components of the flux density can be given as the functions of the dimensional parameters for the doubly salient structure. Thus, the analytical method proposed in this paper can be used to determine the dimensions for the doubly salient structure for improving the working harmonic of the armature magnetic field, which generates output torque on the rotor of the vernier machine.

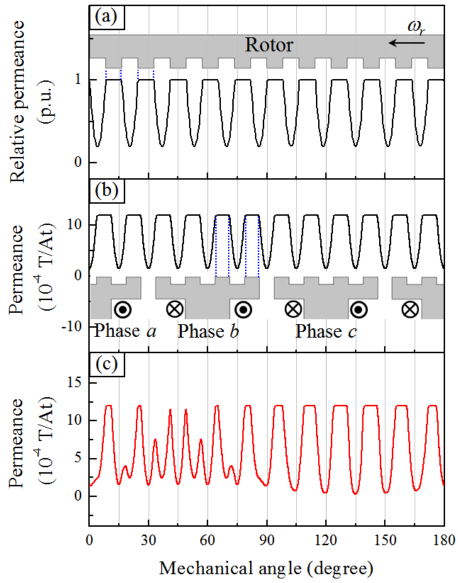

Fig. 2. Air-gap permeance distributions for (a) P_r, (b) P_s, and (c) P

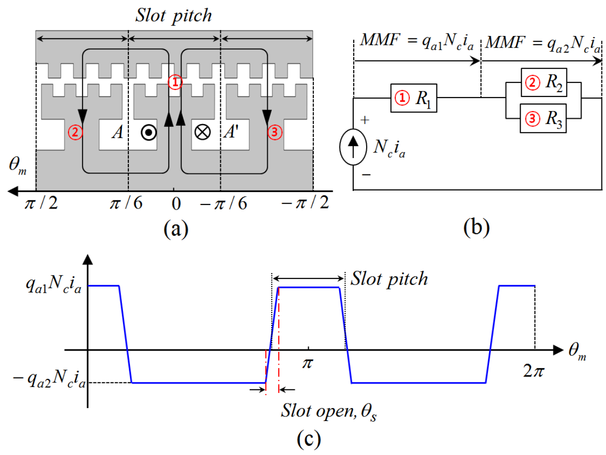

Fig. 3. Winding MMF produced by the winding of the phase a. (a) Magnetic flux path (b) Equivalent magnetic circuit (c) Winding MMF distribution

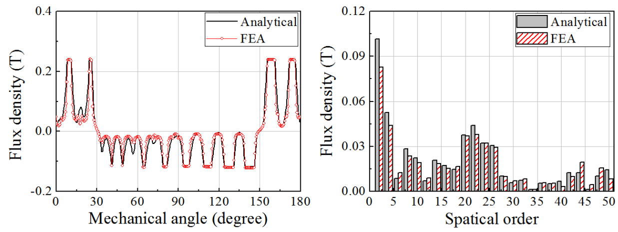

Fig. 4. Flux density distributions and their harmonic components.

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